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Describing the unknown:
Moving toward variable notation and algebraic thinking in kindergarten

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Abstract

This research considers the first year of a longitudinal study in which we are investigating the development of children's algebraic thinking as they take part an early algebra intervention in kindergarten through grade 2. Specifically, this study investigates kindergarteners' understanding of unknown quantities and their representation of these quantities before and after an early algebra intervention. Findings show that as early as kindergarten, many students can develop understandings of unknown values, as well as use variable notation to describe unknowns. Students in the study who did not yet use variable notation were largely able to coordinate assigned quantities accurately, which we believe provides a solid foundation for writing variable expressions to describe unknown quantities.

Introduction

Algebra. A word derived from the Arabic "al-jabr," meaning "the reunion of broken parts" (Oxford English Dictionaries, 2018). Ironically, while the etymology of Algebra describes the goal of Algebra, it also describes an important idea regarding how students can be supported in their learning of algebra. Namely, by teaching algebra as a united strand of thinking throughout students' education rather than only as a stand-alone subject in high school. In other words, math educators must move toward "reuniting the broken parts" that build a strong foundation for algebra learning and doing. With algebra's historic gatekeeper status, now more than ever is there a call for math educators and researchers to take action by ensuring all students experience success with algebra. Fluency in algebra can allow access to higher level mathematics, college, and eventually careers (Blanton, Brizuela, & Stephens, 2016; Stephens, 2005). While we do know many students struggle with algebra in high school, this may be partially because of the way algebra is introduced in schools—as a single, stand-alone subject, rather than as a continuous K-12 strand that is taught throughout the entire mathematics curriculum (Davis, 1985; Kaput, 1998; Stephens, 2005). Although elementary mathematics curricula generally take an arithmetic-based approach, we know that students can think algebraically as early as kindergarten (Brizuela, Blanton, Sawrey, Newman-Owens, & Gardiner, 2015). Research also supports the idea that young children can understand variable quantities and use variable notation (Allen, Brizuela, & Blanton, 2018; Blanton, Brizuela, Gardiner, Sawrey, & Newman-Owens, 2017). Staying true to algebra's etymological beginnings, it is time to "reunite the broken parts" of algebra instruction and use a progressive elementary school algebra

curriculum that prepares students for higher mathematics by laying the foundation for algebraic thinking throughout early childhood.

While students' use and interpretation of variable are more often studied in older grades, mathematics education researchers are just beginning to explore children's understandings of unknown quantities and their ability to represent these quantities using variables. This study investigated kindergarteners' understanding of unknown quantities and their representation of these quantities before and after an early algebra[1] intervention.

Theoretical Framework

Although studies have shown that middle and high school students struggle to correctly use and interpret variable notation (e.g., Booth, 1988; MacGregor & Stacey, 1997), younger students have shown evidence of some facility with variable when given the opportunity to explore this representation around ideas that make sense to them (Brizuela et al., 2015; Brizuela & Earnest, 2008). Once students reach high school algebra and are asked to manipulate variables (something they may not have seen or thought about in school mathematics prior to this point), the opportunity to build a strong foundation for algebra may have been missed. One reason older students may struggle with variable notation is that variables take the form of letters. The use of literary symbols as a stand-in for an unknown value after these symbols have been practiced for years in other non-mathematical contexts (such as reading and writing) and have shown to be a source of confusion (Blanton et al., 2017). Knowing how children are thinking about unknown quantities can help educators build on existing knowledge when introducing the tool of variable notation. Additionally, despite some of the successes we have seen around students' use of variable in the elementary grades, we also have evidence that younger students commonly want to assign numerical quantities to unknowns (Allen et al., 2018; Blanton, Stephens, Knuth, Gardiner, Isler, & Kim, 2015; Brizuela et al., 2015).

These challenges support teaching algebraic thinking early and gradually throughout children's schooling in contexts that make sense to them. Doing so may help prevent confusion with the use of algebraic symbols, as well as provide a solid foundation for more complex problem solving that involves working with unknown quantities.

Method

The study reported on in this paper concerns the first year of a longitudinal study in which we are investigating the development of children's algebraic thinking as they take part an early algebra intervention in kindergarten through grade 2.

Participants

Two classrooms of kindergarteners in a public school in the Northeastern U.S. participated in an early algebra instructional intervention. Ten students from each class participated in interview assessments before and after the intervention. This study reports results from the ten of these kindergarteners whose pre- and post-interviews included a task concerning the interpretation and representation of variable quantities.

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[1] By early algebra, we mean algebraic thinking in the elementary grades.

Intervention

Seventeen intervention lessons were taught during students' kindergarten year between the months of February and May. Two of those lessons (Lessons 12-13) included specific instruction around variables and variable notation. These lessons, which took place during students' regular mathematics instruction time, were taught by our teacher-researcher and covered a range of topics, including identifying and reasoning with even and odd numbers, identifying a missing value in various forms of addition problems, identifying and constructing true equations, working with the equal sign, introducing the idea of a variable to represent an unknown value, and other foundational topics essential to building algebraic understanding. The lessons were designed around the four algebraic thinking practices of generalizing, representing, justifying, and reasoning with mathematical structures and relationships (Blanton, Levi, Crites, & Dougherty, 2011; Kaput, 2008).

Data Collection

Pre- and post-interviews lasting approximately 30 minutes were administered before and after the intervention, which allowed us to see growth in students' understanding of the algebraic topics addressed during our intervention. All interviews were videotaped, and all of students' written work produced during the interviews was collected. Here we focus on one interview task: The *Candy Box* problem (adapted from Carraher, Schliemann, & Schwartz, 2008).

Our version of the *Candy Box* problem involves giving students a small box containing an unknown number of candies that is taped shut. Students can shake the box, but they are not allowed to open the box and count the candies. A known number of candies is then placed on top of the box (1 or 2 additional candies) and students are asked to describe how many candies there are altogether (see appendix, figure 1).

Data Analysis

Codes used to analyze the interviews were developed using a combined top-down and bottom-up approach, with an initial set of codes based on previous work (described in Blanton et al. [2015]), and additional codes added based on common student responses observed after initial viewing of the interviews. All interview responses to the *Candy Box* problem were coded by two members of the research team according to the coding manual in Table 1 (see appendix) and 95% reliability was achieved across the pre- and post-interviews for this task. After this initial round of coding, any disagreements were discussed by both coders after re-watching the relevant portions of the interviews together. Discussion continued until coders reached 100% agreement.

Results

During the pre-interview, when students were shown the box of candy with one or two extra pieces on top and asked what they could say about the number of pieces of candy there were altogether, eight out of ten students identified a specific numerical value. The remaining two students said they could not specify a value, because they did not know how many were in

the box. These types of rigid responses did not allow for follow-up questions, so only one question was asked during the pre-interview.

Three months later during the post-interview when students were asked the same question, of the two students who said they could not know for sure how many candies there were altogether, one now used the variable "*a*" to describe the unknown quantity in the box and the correct related variable expression ("*a* + 2") to describe the number of candies there were altogether. The other student identified a specific numerical value to describe the total number of candies. In total, four kindergarteners used variable notation in the post-interview to describe the number of candies Fran had in the box, and two of these four correctly used a related variable expression to describe the total number of candies (including the one or two pieces on top of the box). While we still had five students using numerical values to describe the number of candies, even these students made progress in using two correctly related numbers (as opposed to the pre-interview, when they just gave a single number as their answer). For example, during the post-interview one student guessed by shaking the box that it sounded like "1000" candies, so when one more candy was placed on top, she "correctly" stated that there were "1001" candies in total. Student responses and codes can be seen in Tables 1 and 2 (appendix).

Discussion and Significance

While the kindergarteners in our study were far from using variable notation to describe unknown quantities in a consistent manner, we do see evidence that these students' understanding of unknowns developed over the four months of the intervention. The number of students who use variable notation increased from zero to four, with five additional students correctly coordinating numerical assignments. This suggests to us that kindergarteners *are* capable of working with variable notation to describe unknown quantities. Additionally, we see that kindergarteners can think abstractly, in that they can imagine how many candies might be in the box and then correctly add a constant to that theorized number.

In future research, we will continue our early algebra intervention with the same students, following them through grade 2. We will continue to see how, with instruction about variable notation and other foundational ideas of algebra, these students learn and form understandings that support their algebraic development.

This study has shown, on a small scale, that kindergarteners can develop their understandings of unknown values, as well as use variable notation to describe unknowns. With the progress we see in just a short intervention program, imagine what would be possible with algebra interwoven as a continuous strand throughout elementary and middle school math, and what a difference this could make towards building a strong foundation for the formalized study of algebra.

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Appendix

Figure 1: Our researcher describes The Candy Box Problem to one of our kindergartener participants.

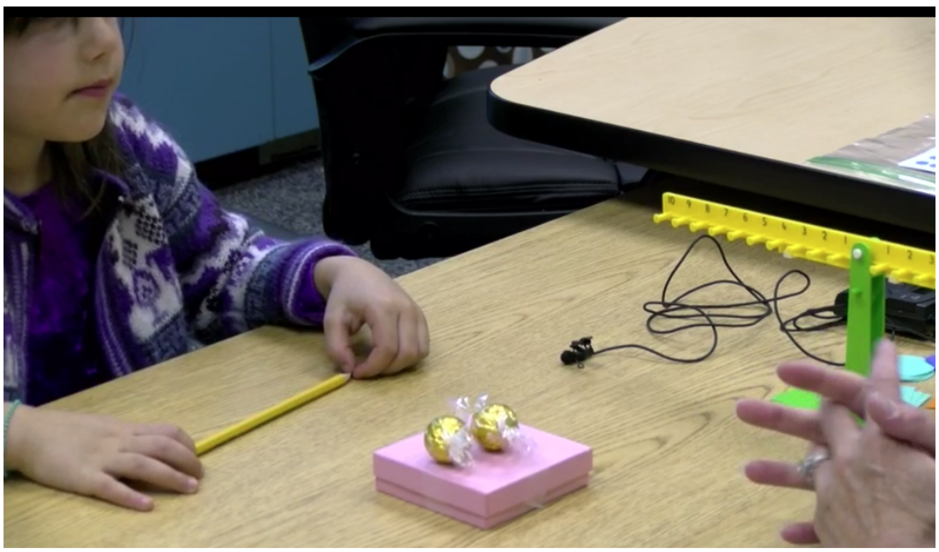

Table 1: Coding procedure for the *Candy Box* problem

Task 5: Developing algebraic expressions	
Interviewer questions	*Codes*
Fran has a box of candy. We don't know how many pieces are in it. Her friend gives her 1 (or 2) **more piece**(s) **of candy.** [Show student a box of candy] **5-a** **What can you tell me about the number of pieces of**	**5-a** **L**—Student assigns a (letter) variable to the unknown amount of candy. **UN**—Student states the number of pieces cannot be known. **V**—Student gives a numerical value for the number of pieces of candy. **OTHER**—Student gives reason different from those above. **5-b** **L—**Student assigns a (letter) variable to the unknown amount

candy Fran has? **<u>5-b</u>** **How would you describe or represent the number of candies in her box?** **<u>5-c</u>** **How would you represent her total number of candies?**	of candy. **UN**—Student states the number of pieces cannot be known. • **V**—Student gives a numerical value for the number of pieces of candy. **OTHER**—Student gives reason different from those above. **<u>5-c</u>** **LR**—Student states a variable expression that correctly coordinates with the response given in part b. • She has $c + 1$ candies (assuming c was the response in part b and there is 1 candy on top of the box) **LN**—Student assigns a letter that is different from the one assigned in part b **L**—Student assigns a (letter) variable to the unknown amount of candy (and did not assign one in part b). **UN**—Student states the number of pieces cannot be known. **V**—Student gives a numerical value for the number of pieces of candy. • I think there are 5 in the box, so she has 7 all together. **OTHER**—Student gives reason different from those above.

Table 2: Student responses when asked to describe how many candies Fran has

How many in the box?	How many total?
p pieces	not sure
3	3 + 1 = 4
She has an *a* amount	*a* + 1
c number of candies	*l*
10	11
1000	1000 + 1
15	16
10	11
z ("because there's a lot")	She has many